

# 國立臺北科技大學 102 學年度碩士班招生考試

系所組別：3620 生化與生醫工程研究所乙組

## 第二節 工程數學 試題

第一頁，共一頁

### 注意事項：

1. 本試題共八題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. (a) Solve the initial-value problem (10%)

$$\frac{dy}{dx} + 2y = f(x) = \begin{cases} 4, & \text{if } 0 \leq x < 1 \\ 0, & \text{if } x \geq 1 \end{cases} \quad y(0) = 1$$

(b) Solve  $\frac{d^2y}{dx^2} - \frac{2y}{x^2} = 0$  (10%)

2. Compute the following convolution (10%)

$$e^{at} * e^{bt}$$

where  $a$  and  $b$  are constants.

3. Find the Laplace transform of the periodic function whose graph is shown in Fig.1. (10%)

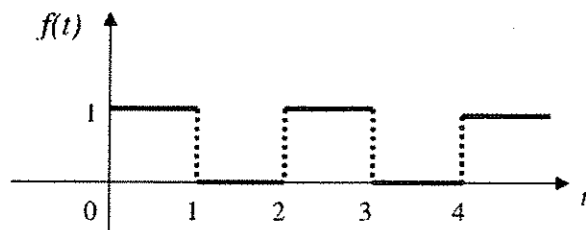


Fig.1

4. Determine the elements of the matrix  $M$  such that  $AMB = C$  when (15%)

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 1 & 3 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 1 \end{bmatrix}$$

5. Let  $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ . Determine nonsingular matrices  $P$  and  $Q$  such that  $PAQ=B$ , where  $B$  is obtained by interchanging the two rows of  $A$  and then adding twice the first column to the third column. (10%)

6. Evaluate the line integral

$$\int_C \frac{dx + dy}{|x| + |y|}$$

where  $C$  is the square with vertices  $(1,0)$ ,  $(0,1)$ ,  $(-1,0)$ , and  $(0,-1)$ , traversed once in a counterclockwise direction. (10%)

7. Let

$$g(x) = \begin{cases} 0 & -\pi \leq x < -\frac{\pi}{2} \\ 1 & -\frac{\pi}{2} \leq x \leq 0 \\ 0 & 0 < x \leq \pi \end{cases}$$

Determine the complex Fourier series of  $g(x)$ . (10%)

8. Determine that solution of the partial differential equation

$$5 \frac{\partial f(x, y)}{\partial x} - 2 \frac{\partial f(x, y)}{\partial y} = 0$$

which satisfies the conditions  $f(0,0)=0$  and  $\frac{\partial}{\partial x} f(x,0) = e^x$  for all  $x$ . (15%)