國 立 臺 北 科 技 大 學

九十二學年度機電科技研究所博士班入學考試

控制系統（電機組）試題

填 准 考 證 號 碼　　　　　　　　　　　　　　　　第一頁　共一頁

注意事項：

1. 本試題共【五】題，配分共 100 分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在答案卷之答案欄內，否則不予計分。

1. Explain the following terms.
   (a) Gain/Phase margin (5%)　(b) Separation principle (5%)
   (c) Linear quadratic regulation (5%)　(d) Equilibrium point (5%)

2. For the dynamic system
   $$\dot{x}(t) = Ax(t) + Bu(t)$$
   $$y(t) = Cx(t) + Du(t),$$
   where $x(t)$ is the state vector and $y(t)$ is the output. Prove that the state feedback law $u(t) = -Kx(t) + r$ does not affect the controllability. (20%)

3. Show that the transformation $x = P\bar{x}$ converts the system $\dot{x} = Ax + Bu$ into

$$\dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}u \quad \text{where} \quad \bar{A} = P^{-1}AP = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ -\alpha_n & -\alpha_{n-1} & -\alpha_{n-2} & \cdots & -\alpha_2 & -\alpha_1 \end{bmatrix},$$

$$\bar{B} = P^{-1}B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \text{ and } P = \begin{bmatrix} B & AB & \cdots & A^{n-1}B \end{bmatrix} \begin{bmatrix} \alpha_{n-1} & \alpha_{n-2} & \cdots & \alpha_1 & 1 \\ \alpha_{n-2} & \alpha_{n-3} & \cdots & 1 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ \alpha_1 & 1 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \end{bmatrix}. \text{ (20\%)}$$

4. For the system

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t),$$
$$y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} x(t)$$

   (a) Find the zero-input response of $x(t)$ with the initial state $x_0 = x(0)$. (10%)
   (b) Design an observer that reconstructs the states and pick the observer roots to be at $s = -5 \pm j5$. (5%)
   (c) Design a state feedback law $u(t) = -Kx(t) + r$ so that the closed-loop poles are at $s = -1 \pm j1$. (5%)

5. Consider the Van der Pol oscillator described by

$$\dot{x}_1 = x_2 - x_1(x_1^2 + x_2^2)$$
$$\dot{x}_2 = -x_1 - x_2(x_1^2 + x_2^2).$$

   (a) Find the equilibrium point. (10%)
   (b) Examine the stability using Lyapunov approach. (Hint: Consider the Lyapunov candidate $V(x) = x_1^2 + x_2^2$.) (10%)