

國立臺北科技大學

九十三學年度商業自動化與管理研究所入學考試

線性代數試題

填准考證號碼

第一頁 共一頁

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注意事項：

1. 本試題共三題，配分共 100 分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在答案卷之答案欄內，否則不予計分。

A、Mark each statement True (O) or False (X). (80 points.) (答錯倒扣兩分)

(Each question 4 points, if your answer is wrong, deduct 6 points.)

1. A change-of-coordinates matrix is always invertible.
2. The nonpivot columns of a matrix are always linearly dependent.
3. If \mathbf{B} is obtained from a matrix \mathbf{A} by several elementary row operations, then $\text{rank } \mathbf{B} = \text{rank } \mathbf{A}$.
4. If \mathbf{A} is $m \times n$ and $\text{rank } \mathbf{A} = m$, then the linear transformation $\mathbf{x} \rightarrow \mathbf{A}\mathbf{x}$ is one-to-one.
5. If an $m \times n$ matrix \mathbf{A} is row equivalent to an echelon matrix \mathbf{U} , and if \mathbf{U} has k nonzero rows, then the dimension of the solution space of $\mathbf{A}\mathbf{x} = \mathbf{0}$ is $m - k$.
6. If \mathbf{A} is a 3×3 matrix with three pivot positions, there exist elementary matrices $\mathbf{E}_1, \dots, \mathbf{E}_p$, such that $\mathbf{E}_p \cdots \mathbf{E}_1 \mathbf{A} = \mathbf{I}$.
7. An elementary $n \times n$ matrix has either n or $n + 1$ nonzero entries.
8. If \mathbf{A} and \mathbf{B} are square and invertible, then \mathbf{AB} is invertible, and $(\mathbf{AB})^{-1} = \mathbf{A}^{-1}\mathbf{B}^{-1}$.
9. If $\mathbf{AB} = \mathbf{BA}$ and if \mathbf{A} is invertible, then $\mathbf{A}^{-1}\mathbf{B} = \mathbf{BA}^{-1}$.
10. If $\mathbf{BC} = \mathbf{BD}$, then $\mathbf{C} = \mathbf{D}$.
11. Suppose $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are nonzero vectors in \mathbf{R}^5 , \mathbf{v} is not a multiple of \mathbf{u} , and \mathbf{w} is not a linear combination of \mathbf{u} and \mathbf{v} . Then $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent.
12. In some case, it is possible for four vectors to span \mathbf{R}^5 .
13. If an augmented matrix $[\mathbf{A} \ \mathbf{b}]$ can be transformed by elementary row operations into reduced echelon form, then the equation $\mathbf{A}\mathbf{x} = \mathbf{b}$ is consistent.

14. If a system $\mathbf{Ax} = \mathbf{b}$ has more than one solution, then so does the system $\mathbf{Ax} = \mathbf{0}$.
15. If an augmented matrix $[\mathbf{A} \ \mathbf{b}]$ is transformed into $[\mathbf{C} \ \mathbf{d}]$ by elementary row operations, then the equations $\mathbf{Ax} = \mathbf{b}$ and $\mathbf{Cx} = \mathbf{d}$ have exactly the same solution sets.
16. Every matrix is row equivalent to a unique matrix in echelon form.
17. If a system of linear equations has no free variables, then it has a unique solution.
18. An under constrained linear system cannot have a unique solution.
19. A system of linear equations has infinitely many solutions if and only if at least one column in the coefficient matrix does not contain a pivot position.
20. If two augmented matrices are row equivalent, then the associated systems of linear equations have the same solution set.

B · Use the following hypotheses to determine the best conclusion(s), if any. (10 points)

I will study enough if I go out only on weekends.

I can pass all my courses if I study enough.

If I give up drinking, then I'll go out only on weekends.

If my phony ID is taken away, then I'll stop drinking.

I failed at least one course.

C · Construct a truth table for $[p \vee (q \wedge \sim p)] \rightarrow r$ (10 points)