

國立臺北科技大學九十五學年度碩士班招生考試

系所組別：1310、1320、1330 車輛工程系碩士班甲乙丙組

第二節 工程數學 試題

填 准 考 證 號 碼

--	--	--	--	--	--	--	--

第一頁 共一頁

注意事項：

1. 本試題共 4 題，配分每題 25 分，共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

一、 $y_1(x)$ is a known solution of an ordinary differential equation $y'' + m(x)y' + n(x)y = 0$.

1. Prove that the second solution $y_2(x)$ can be found by $y_2(x) = p(x)y_1(x)$, where

$$p(x) = \int \exp\left(-\int \frac{2y_1'(x) + m(x)y_1(x)}{y_1(x)} dx\right) dx \quad (10 \text{ 分})$$

2. That differential equation has parameters of $m(x) = -2$ and $n(x) = 1$ with initial conditions $y(0) = 2$ and $y'(0) = 1$. Besides, an external force $F(x) = 3x + 25\sin(3x)$ applies to this system to make this system nonhomogeneous. Find the general solution for this nonhomogeneous system. (15 分)

二、A cart (mass $m=1$) running on a frictionless surface is connected to a wall by a linear

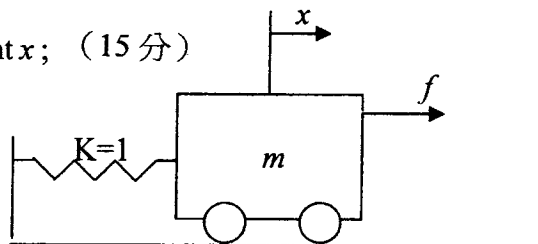
spring with spring constant 16. There is a force $f(x) = \begin{cases} \cos 4t, & \text{for } 0 \leq t < \pi \\ 0, & \text{for } t \geq \pi \end{cases}$ applied

to this cart. This cart is pushed forward with a positive velocity $v=1$ while the cart is released from its equilibrium position at time $t=0$.

1. find the Laplace transform for the cart displacement x ; (15 分)

2. solve the cart displacement $x(t)$ at time $t=10$

using Laplace transform method. (10 分)



三、A system has dynamics shown as $\dot{X} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -2.5 & -3 & 3 \\ -4.5 & -4 & 6 \\ -6 & -6 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = AX$, where

x_1, x_2 and x_3 are 3 displacements for this system. The matrix A shows that the three displacements are coupled together. Such a coupled system representation can be transferred into an uncoupled system representation.

1. Find the uncoupled system representation; (10分)
2. Write down the detailed transferring process from coupled system representation to uncoupled system representation. (15分)

四、A function $f(x)$ has the form as $f(x) = \begin{cases} x^2, & \text{for } 0 \leq x < 1 \\ 1, & \text{for } 1 \leq x < 4 \end{cases}$.

1. Find the Fourier cosine series and the Fourier sine series for this function. (15分)
2. Determine the sum of each series. (10分)